# Test 1A, Math of Democracy
## Dr. Adam Graham-Squire, Fall 2018

Name: _____ Key_____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____
(*signature*)

# DIRECTIONS

1. Don't panic.

2. **Show/explain all of your work**. A correct answer with insufficient work will lose points.

3. Read each question carefully, and make sure you answer the the question that is asked. If the question asks for an explanation, make sure you give one.

4. Clearly indicate your answer.

5. Calculators are allowed on this test, but any other technology (cell phones, computers, etc) is NOT allowed.

6. Make sure you sign the pledge.

7. Number of questions = 4. Total Points = 20.

1. (5 points) Consider the following preference schedule:

| Number of voters | 3 | 1 | 1 | 4 | 4 |
|---|---|---|---|---|---|
| 1st choice | A | A | A | B | D |
| 2nd choice | B | C | D | A | C |
| 3rd choice | C | D | C | C | B |
| 4th choice | D | B | B | D | A |

(a) Use the Pairwise Comparison method to find the winner of the election (if two candidates are tied in a head-to-head, give each of them a half-point). Briefly show/explain your work.

B beats A   8-5          A beats C          C beats D

B beats C   7-6          A beats D                        ✓ 0.5

B beats D   7-6

⮡ B is Condorcet Winner, wins the P.C.

(b) Now suppose the the first column of voters decided to NOT show up for the election. The preference schedule now looks like:

| Number of voters | 1 | 1 | 4 | 4 |
|---|---|---|---|---|
| 1st choice | A | A | B | D |
| 2nd choice | C | D | A | C |
| 3rd choice | D | C | C | B |
| 4th choice | B | B | D | A |

Run the Pairwise Comparison election on the new preference schedule. What, if anything, does your result tell you about Pairwise Comparison and a Fairness Criterion? Explain your answer.

A beats   C and D        6-4  ⟹  A gets 2 points

B beats A    but    loses to   C and D    6-4

                         ⟹ B get 1 point ✓ 0.5

C and D tie

⮡ ⟹ both get 1.5 points

✓ So  A  wins!  Those 3 voters not showing up
✓ made their prefered candidate (A) with over B.
Shows that P.C. fails the No-show criteria!

2. (5 points) Will the following election, under the Instant Runoff method, demonstrate a monotonicity anomaly?

| Number of voters | 22 | 8 | 15 | 17 |
| --- | --- | --- | --- | --- |
| 1st choice | A | B | B | C |
| 2nd choice | B | A | C | A |
| 3rd choice | C | C | A | B |

→ C drops out, voter to A and A wins ✓

- If *so*, explain what ballots would need to change, and in what way, to cause the monotonicity anomaly.

- If *not*, explain how changing votes in *any* of the columns could never cause a monotonicity anomaly.

Note: It is fine to reference the criteria for the presence of a monotonicity anomaly in a three-candidate IRV election, but for a complete answer you should actually modify the preference schedule in some way to support your conclusions.

Total # of voter is 62, $\frac{17}{62} = 0.27 > 25\%$ ✓

C beati A in heads-to-head 32−30 ✓

⇒ Must be a mono anomaly!

Move 7 BAC votes to ABC, get ✓

$$\frac{29}{\begin{matrix} A \\ B \\ C \end{matrix}} \quad \frac{1}{\begin{matrix} B \\ A \\ C \end{matrix}} \quad \frac{15}{\begin{matrix} B \\ C \\ A \end{matrix}} \quad \frac{17}{\begin{matrix} C \\ A \\ B \end{matrix}}$$

B drops out first, gives 15 votes to C, who wins! ✓

A got 7 more votes and *lost* the election

✓ ⇒ Monotonicity anomaly.

3. (5 points) Below are listed 4 pairs of voting methods and fairness criteria. Choose *two* of the pairs and explain why that particular voting method either *satisfies* or *violates* that particular criteria. Of the two you choose, one of them must be a proof/explanation for how the method satisfies, and the other must be a proof/explanation of how the method violates. If you do more than 2, I will score all of them and give you the highest of the appropriate scores.

- Range voting and Independence of Irrelevant Alternatives criterion
- Sequential Pairwise voting and Condorcet Winner criterion
- Coombs Method and ~~Monotonicity~~ IIA criterion
- Top-two runoff and No-show criterion

*(handwritten annotation, top right):* → satisfies b/c a Cond. winner beats everyone in a head-to-head, so th will win each sequential pairwise vote as well, and always win, no matter t agenda.

. Range voting <u>satisfies</u> IIA, because each candidate is sored seperately. Removing a losing candidate does not change the scores of other candidates b/c each vote gives seperate points to each candidate. So the winning candidate will still have the most (after removal of loser) b/c no ones points will change.

- Coombs and ~~Mono~~ IIA: <u>fails</u>   Consider this election

*(annotation):* → A drops in first round, then B, then C so D wins Coombs

if 3 BACD don't show then A wins

| 24 | 25 | ~~43~~ 3 | 16 | 16 | 16 |
|----|----|----|----|----|----|
| A  | A  | B  | B  | C  | D  |
| B  | C  | A  | D  | B  | C  |
| D  | D  | C  | C  | D  | B  |
| C  | B  | D  | A  | A  | A  |

*(annotation, right):* with C and D gone, A wins Top-two runoff. Remov 2 BCDA though, makes runoff b/t

- Top two and No-show:   Fails

| 10 | 9 | 8 | 7 |
|----|----|----|----|
| A  | B  | C  | D  |
| B  | C  | D  | C  |
|    | C  | D  | C  |
|    | D  | A  | A  |

4. (5 points) (a) Of all of the voting methods we have looked at so far, which one do you think is the best? Briefly explain why it is better than the other methods.

(b) Of all of the fairness criteria we have seen so far, which one do you think is *most* important for an election to have? Explain (briefly) why that particular criteria is so important for a fair election.

Many different answers!

Scoring: Need to include something beyond just what is on Description of Methods. Best is comparison to other Methods,

(a) I think Pairwise Comparison is the best b/c [Criteri]
it encourages people to fully rank candidates, and
if a candidate beats a lot of others, then
they should win the election. If it ends in a
tie, need a backup method like Borda. Borda count
fails Majority, which I think is very important. Instant
Runoff fails a *lot* of criteria (Monotonicity, etc) b/c of
the changes in deport order. Top-two and plurality don't
take enough information into consideration.

So, ∴ is one-cut...

(b) The Condorcet Winner criterion is my most
important, b/c I think most people acknowledge that
if someone can beat everyone else, they should be
the winner. Majority is also important, but if you
have Condorcet then Majority is guaranteed. So many
methods fail IIA that it doesn't seem as worth it.
Monotonicity and No-show seem to be more esoteric
because they only seem to fail in strange
situations in which the election is very contested.

**Extra Credit** (1 point) Under any of the voting methods we have seen, is it possible to have a candidate who is not ranked first in *any* ballots, but still wins the election? If so, explain what the method is and give an example of how it could happen. If not, explain why it is NOT possible for at least 3 of the voting methods.

**Yes**     Pairwise comparison:

| 27 | 26 | 24 | 23 |
|----|----|----|----|
| A  | B  | C  | D  |
| E  | E  | E  | E  |
| B  | A  | D  | A  |
| C  | C  | A  | B  |
| D  | D  | B  | C  |

E beats all other candidates in a head to head

A: 27   to   $\frac{E}{73}$

B: 26   to   74

C: 24   to   76

D: 23   to   77

So E is Condorcet winner and wins Pairwise but is __not__ ranked 1st in any ballot!